Heart Instantaneous Frequency (HIF): An Alternative Approach to Extract Heart Rate Variability

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Abstract—Our study focuses on a new method of estimating the heart rate variability (HRV) which does not require the use of electrocardiogram (ECG) R-wave detection. Contrary to the R-wave detection method which requires a sampling frequency higher than 100 Hz, the one proposed here can be used to calculate the HRV from an ECG signal sampled at a frequency of approximately 5 Hz with a relative mean error of 0.03. This new method is based on extracting the instantaneous fundamental frequency from the ECG. The method could be efficiently used to extract the HRV from an ECG measured for healthy subjects performing an exercise in which the HRV increases linearly with time, and for subjects with respiratory and cardiac problems. The overall error decreased as we low-pass filtered the HRV with lower cut-off frequencies. Moreover, it was shown that the method could be efficiently used to calculate the HRV from blood pressure measurements and to be robust to noise.

Index Terms—Analytic signal, heart rate variability, instantaneous frequency.

I. INTRODUCTION

THE SPONTANEOUS rhythmic activity of the heart is a well known phenomenon. This repeating behavior can be easily accessed through the heart rate variability (HRV) by electrocardiogram (ECG) measurement. HRV characterizes for homeostatic purposes the fluctuations, with distinct periods, of the cardiac system caused by different intrinsic regulation mechanisms. This regulation occurs in order to maintain the cardiovascular system working to preserve life, independently of conditions.

Moreover, HRV is a useful tool for understanding the status of the autonomic nervous system (ANS). Spontaneous heart beat fluctuations are mainly due to interactions between cardiac pacemaker cells and the sympathetic and parasympathetic systems. The HRV results from the balance between both components [1]. Accordingly, the sympathetic and parasympathetic systems modulate the cardiac activity.

Usually, an ECG is measured on the subject’s thorax surface and stored to later analyze the HRV. However, the frequency at which the ECG is sampled is considerably higher than the one required for analysis of the HRV. For example, Holter systems in which the ECG is measured for periods up to 24 h use a sampling frequency of 128 Hz. On the other hand, for analysis of the HRV during exercise, a frequency as high as 500 Hz is required. However, the required spectral response of the HRV is limited to 0.5 Hz, which implies that for measuring HRV it would be necessary only to sample the signal at 1 Hz!

One solution would be to reduce the ECG sampling rate. However, Merri et al. [9] showed that the accuracy in the calculation of HRV based on the R-wave detection method decreases exponentially with decreasing sampling frequency.

In this paper, we propose an algorithm for measuring HRV that reduces the required ECG sampling frequency to around 5 Hz for an ECG extracted during resting and to 20 Hz for one extracted during exercise. We designate the HRV measured in this way as heart instantaneous frequency (HIF). The fundamental concept underlying the calculation of HIF is to take advantage of the repetitive behavior, or quasiperiodicity of the heartbeat. Thus, in this paper, we estimate the fundamental frequency from the ECG spectral response, and show that this frequency is equal to the HRV with a small degree of error. Moreover, we show that HRV can be estimated accurately from the HIF when calculated from other types of cardiac signals, such as blood pressure. Finally, we study the robustness of the new method to calculate HRV using a noise corrupted ECG.

Another problem is that HRV is a nonstationary signal, which means that its statistical properties change with time. This characteristic of HRV may pose problems for the conventional methods used to estimate instantaneous frequency. To overcome this difficulty we use wavelets which are efficient tools for nonstationary signal analysis.

Extracting the instantaneous frequency of signals is largely used in speech signal processing. One such method was proposed by Kawahara [7], including one application of the method to HRV measurement which we used in a previous study [3]. However, in this paper, we use the instantaneous frequency framework specifically to extract the HRV.

II. OUTLINE OF THE METHOD

The electrocardiogram has regular peaks known as R-waves, which are generated by the heart beating. The HRV is inversely proportional to the time differences between two consecutive R-waves. More precisely, let us define a time series composed of the time differences (in seconds) between two consecutive R-waves as \( x = [t_1, t_2, \ldots, t_n]^T \). The HRV is understood as \( \text{HRV} = \{00/t_1, 00/t_2, \ldots, 00/t_n\}^T \), and is given in beats/min.

In our reasoning, let us first assume that heart beats with a perfect periodic timing, thus \( t_1 = t_2 = \cdots = t_n = T \). Then, the ECG signal will have its fundamental frequency given by
\[ \omega_0 = \frac{2\pi}{T}, \] where \( T \) is the period. In this case, the power spectrum of the ECG signal, \( z(t) \), would exhibit peaks at \([\omega_0, 2\omega_0, 3\omega_0, \ldots]\). In other words, \( z(t) \) would be composed of a periodic signal with a fundamental frequency \( \omega_0 \) and an infinite number of harmonics. To estimate \( \omega_0 \) from the signal spectral response, however, it is not necessary to have the entire spectral response for \( z(t) \). If we limit the digitalized \( z(k) \), where \( k \) is the sampling time, to a sampling frequency higher than \( 2\omega_0 \), we can still estimate \( \omega_0 \) using at least two methods: 1) by counting the number of times that the signal crosses zero or 2) by analysis of the \( z(t) \) power spectrum. In this paper, we used the latter method.

The heart beats at a rate which is not constant. Thus, to implement our ideas, we used the concept of modulation. We modeled the heart as if it had a fundamental frequency which is altered in time by different factors. For example, it is known that the heart rate fluctuates in response to changes in respiration. This fluctuation is called respiratory sinus arrhythmia (RSA). Therefore, we classify the RSA as a frequency modulation occurring in the cardiac signal, which makes the heart rate (or heart frequency) vary according to changes in respiration.

Frequency modulation reported in the signal processing literature (e.g., [4]) led us to the possibility of using the concept of instantaneous frequency. For a given signal \( s(t) \), the instantaneous angular frequency \( \omega(t) \) is calculated as

\[ \omega(t) = \frac{d\phi(t)}{dt}, \quad \psi(t) = \arctan\left( -\frac{H[s(t)]}{s(t)} \right) \] (1)

where \( H[s(t)] \) is the Hilbert transform of the signal \( s(t) \).

Therefore, in this paper, we estimate the instantaneous frequency of the ECG signal and compare it with the actual HRV extracted using the classical R-wave detection method.

III. HEART INSTANTANEOUS FREQUENCY

We termed the HRV estimated using the ECG spectral response the HIF. HIF is extracted from the spectral response of the ECG. As this ECG signal \( z(t) \) exhibits multiple harmonics, it has to be filtered in an effective way so that a new signal \( s(t) \), having only the ECG fundamental frequency remains, from which the instantaneous frequency is extracted. As this fundamental frequency varies with time, the filter characteristics should vary accordingly. Indeed, the steps required to carry out the extraction of HIF are listed in Fig. 4. Below, we outline the theoretical framework.

First, the extraction involves the estimation of the spectrogram [see Fig. 1(a) for one example]. The spectrogram for signal \( z(t) \) is defined as

\[ P(t, f) = \frac{1}{2\pi} \left| \int e^{-j2\pi f \tau} z(\tau) h(\tau - t) d\tau \right|^2 \] (2)

where \( h(\tau - t) \) is a window function which slides along \( z(t) \).

Then, we find the frequency value corresponding to the maximum of \( P(t, f) \) at each time point in a given frequency range. We termed this value the driver \( \delta(t) \), and it is given by

\[ \delta(t) = \arg \max_f [P(t, f)]^{(t-\tau)\alpha}, \] (3)

We define \( \alpha \) as the frequency value that limits the search range, and \( \delta(t) \) as the driver value at previous time \( t \). A value of \( \alpha \) of around 0.3–0.5 Hz yielded accurate estimations.

Generally speaking, (3) is used to express the fact that the algorithm searches for the maximum \( P(t, f) \) at each time point \( t \) along the frequency axis \( f \) which is bounded to interval \([\delta(t) - \alpha, \delta(t) + \alpha]\).

We can see that (3) is a recursive equation. The current value of \( \delta(t) \) is a nonlinear function of \( \max_f [P(t, f)] \) and previous \( \delta(t) \). This is because if we took the maximum value of \( \max_f [P(t, f)] \) at each time, there could be a shift of attention, e.g., at one time point, the maximum of the spectrogram could be located at the fundamental frequency, but in the next time, it could shift to the first frequency harmonic of the signal, as it occurs when we used the method reported in [7]. Thus, using the recursive equation, we force the current value \( \delta(t) \) to be close to the previous one, because we know that the HRV does not change abruptly, as in the case of the voice pitch [7].

Finally, we calculate the instantaneous frequency using a bandpass filter around a central frequency given at each time point by the driver. In particular, we use wavelets to construct the filter. The basic wavelet is a slightly modified Gabor function, which is localized in both time and frequency domains. The modification was carried out in order to shift the spectral response of the filter to the central frequency. Thus, the basic wavelet is given by

\[ \psi(t) = \frac{1}{2\pi} \frac{d}{dt} \left[ \exp\left( -\pi \left( \frac{\delta(t) t}{2} \right)^2 \right) \cos \left( 2\pi t \int_{\Omega} \delta(\tau) d\tau \right) \right], \] (4)

where \( \Omega \) is a short time interval. The signal filtered in this interval is given by

\[ s_\Omega(t) = \int_{\Omega} \int \delta(\tau) \psi(t - \tau) d\tau. \] (5)

The HIF is then calculated by substituting (5) into (1).

IV. RESULTS

We applied the above method to electrocardiograms extracted for a group of four subjects with sleep apnoea; for another group of four subjects with cardiac arrhythmia, obtained from the MIT-BIH sleep apnoea and arrhythmia libraries, respectively; and for another group of four subjects during exercise. In the latter case, the subjects were asked to pedal on a bicycle ergometer, the load of which increased linearly with time until the subjects were completely exhausted. Data intervals varied from 15 to 20 min in length. The ECG for the sleep apnoea group was originally sampled at 250 Hz, for the arrhythmia group at 360 Hz, and for the exercise group at 500 Hz. We down-sampled and low-pass filtered them to frequencies of 5, 6, and 20 Hz, respectively, and then applied the technique proposed above.

We used zero padding to handle border distortions caused by the use of wavelets.

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The difference in sampling frequencies used is due to the fact that during exercise the fundamental frequency of an ECG, in contrast to during the resting condition, has less energy than the first harmonic frequency and, therefore, is more susceptible to being affected by respiratory artifacts, of which there are many during exercise. On the other hand, the fundamental frequency during resting conditions usually varies between 1~1.5 Hz, and thus a sampling frequency of 5 or 6 Hz is sufficient.

For the sake of comparison, we also obtained the HRV (from original signals) using a threshold method to detect R-waves. Moreover, the HIF was interpolated to the frequency of the original ECG signal so that the HIF could be obtained at the same points at which the ECG R-waves occurred. Fig. 1 shows an HRV compared with the proposed HIF obtained for a subject with sleep apnoea.

We also calculated the relative error between the two methods using the following:

\[ \varepsilon = \sqrt{\frac{1}{N} \sum_{j=1}^{N} \left[ \frac{HIF(j) - HRV(j)}{HRV(j)} \right]^2} \]
where $N$ is the total number of beats. The results are given in Table I. As one can notice, (6) gives the relative standard deviation of the error, with minimum and maximum being zero and one, respectively.

### A. Measurements from Blood Pressure

We used our algorithm to measure the HIF from a blood pressure (BP) signal obtained from the MIT-BIH sleep apnoea library. We took intervals from the BP signal measured for four different subjects with each interval consisting of 5-min measurements. The error was calculated using (6). The results are shown in Table II.

### B. Robustness to White Noise

We also checked the robustness of the method to white noise using the HIF to calculate the HRV from an ECG corrupted with white Gaussian noise added at different levels. To achieve this, the ECG signal was standardized to have zero mean and unity variance. The noise (0, 1) was multiplied by different factors: zero, 0.5, 1, 1.5, and 2, and added to the clean signal. The calculated error is shown in Table III.

In Fig. 2, one can see how the addition of white noise scaled to a factor of 2.0 (as given in Table III) completely destroys the ECG shape and, therefore, the R-wave cannot be detected. In this case, the proposed method had an error of 0.065.

### V. Discussion

In the analysis of the HRV signal, three major regions appear in the power spectrum of the signal. The high-frequency region (HF, 0.14–0.40 Hz), which is normally related to respiratory activity; the low frequency region (LF, 0.04–0.15 Hz); and the very low frequency region (VLF, below 0.04 Hz). Recently, there has been an increased interest in the VLF region [10], [11]. To study this region, usually 24 h of ECG data is required, which means that if the R–R interval method to calculate HRV were used, there could be a huge amount of wasted memory.

In this particular case, the method proposed here works efficiently. To demonstrate this efficiency, we passed both signals (HIF and HRV) through a low-pass filter with a cutoff frequency of 0.05 beat$^{-1}$, which is ten times higher than the one used by Peng et al. [10], and we obtained the error shown in Table IV.

On the other hand, it is important to notice that the analysis of the various spectral regions of the HRV works in the same way to HIF. For example, if one wants to study only the HF region, one should just carry out a Fourier transform on the HIF signal and analyze the higher frequency of the result. A benefit in this case is that one will have direct access to the information in a Hz basis, instead of the usual beat$^{-1}$. In fact, in the R–R method, if one wants to use a s/Hz basis, one is forced to carry out an interpolation, which usually introduces unwanted distortions.

Another advantage of the proposed method is that it can be used to calculate the HRV during exercise. Usually, calculation of the HRV requires a high ECG accuracy in this particular case. Thus, instead of using a sampling frequency of approximately

![Fig. 2. Example of two ECG signals used to check robustness to noise. (a) ECG original signal. (b) ECG corrupted by white Gaussian noise. Notice that the R–R interval information has been completely lost in this last case.](image-url)
Fig. 3. The power spectrum (PS) for the two last signals in Fig. 1. (a) PS for HIF. (b) PS for the classical HRV. (c) The difference between the two previous spectra. Notice that it is flat, resembling a white noise spectral distribution. The horizontal axis is the normalized frequency.

100 Hz, a much higher frequency is required. This is due to the accuracy required for detection of the R-waves. For example, to calculate the HRV with an accuracy of 10 ms, a sampling frequency of 100 Hz is required. This degree of accuracy may be sufficient if the heart is beating at the usual 70 beats/min during resting conditions. However, if it changes to 180 beats/min, which usually happens in an exhaustion test experiment, a higher sampling frequency is required, usually above 500 Hz. On the other hand, for measuring the HIF during exercise, a sampling frequency higher than 5 Hz may be required, due to: 1) the increase in heart rate which usually goes up to 1.5 Hz during rest, but during exercise, it can reach 3 Hz; 2) augmentation of the signal energy such as the respiration at the fundamental frequency range, which forces an estimation of the first frequency harmonic instead of the fundamental frequency. Even taking this into account, the proposed method is still advantageous when compared with the conventional one.

Interestingly, upon examination of the difference between the power spectra of the HIF and HRV as in Fig. 3, we observed a flat spectrum, resembling that of a white noise, which demonstrates the efficiency of the method.

In our case, the time-frequency resolution of the spectrogram is not a critical issue, because it serves no more to give a rough estimation of the instantaneous frequency. That is why we introduced the concept of the driver. Indeed, the actual resolution for the instantaneous frequency is given by the wavelet filter. Some words are also necessary in this matter. The advantage of using a wavelet filter compared with a classical one is that it has a better time frequency resolution. In our algorithm, we introduced a Gaussian decay in (6) given by the exponential term. This gives a symmetry both in time and frequency which does not occur in classical filters. Moreover, the filter works carrying out an automatic averaging over the driver, which in other words works as if it was an adaptive filter with the central frequency changing in time.

It is also important to emphasize that the method performed efficiently when calculating the HRV for blood pressure data. We believe that it will also perform efficiently when calculating the HRV from data pertaining to other cardiac related events. However, it is important to note that extracting the HRV from signals other than the ECG might sometimes show discrepancies [6]. Therefore, it can be expected that some cardiovascular complications will not be identified when the HRV is measured from blood pressure data, whereas they can be when the HRV is measured from an ECG.

Moreover, the method used was efficient in the calculation of the HRV when the ECG was corrupted by random artifacts. This is clear from the relative error of only 0.065 obtained when the method was used to calculate the HRV from an ECG signal corrupted by white noise such as that shown in Fig. 2.

From the above results obtained by comparing HIF with HRV, we can see that the HIF is an efficient estimator for HRV. In addition, there are other advantages of the proposed method: HRV is calculated as a time signal which means that it can be easily related to other physiological signals; the detection of R-waves is not required; not much memory is required when measuring the ECG; it can be used to calculate the HRV from data pertaining to other cardiac events, such as blood pressure; it is robust to noise.

Moreover, calculating the HIF from a 20-min-long ECG took less than 10 s to complete all the operations in a Sparc20 workstation, using MATLAB.

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